

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 9

It is used to solve such an equation in which variables can be separated completely. For eg: $vdv = x dy$ can be solved as $\frac{dx}{x} = \frac{dy}{y}$. Integrating both sides $\log x = \log y + \log c \Rightarrow \frac{x}{y} = c \Rightarrow x = cy$ is the solution.

The order of a Differential Equations representing a family of curves is same as the number of arbitrary constants present in the equation corresponding to the family of curves. For eg: Let the family of curves be $y = mx$, $m = \text{constant}$, then, $y' = m$
 $y = y'x \Rightarrow y = \frac{dy}{dx}x \Rightarrow x \frac{dy}{dx} - y = 0$.

A Differential Equation which can be expressed in the form $\frac{dy}{dx} = f(x, y)$ or $\frac{dx}{dy} = g(x, y)$, where, $f(x, y)$ and $g(x, y)$ are homogeneous functions of degree zero is called a homogeneous Differential Equation
 For eg: $(x^2 + xy)dy = (x^2 + y^2)dx$
 To solve this, we substitute $\frac{y}{x} = v \Rightarrow y = vx$.

A Differential Equation of the form $\frac{dy}{dx} + Py = Q$, where P, Q are constants or functions of 'x' only is called a first order linear Differential Equations its solution is $y e^{\int P dx} = \int Q \cdot e^{\int P dx} dx + c$. For eg: $\frac{dy}{dx} + 3y = 2x$ has solution $y e^{\int 3 dx} = \int 2x \cdot e^{\int 3 dx} dx + c \Rightarrow y e^{3x} = 2 \int x e^{3x} + c$.

To form a Differential Equation from a given function, we differentiate the function successively as many times as the no. of arbitrary constants in the given function, and then eliminate the arbitrary constants. For eg: Let the function be $y = ax + b$, then we have to differentiate it two times, since there are 2 arbitrary constants a and b . $\therefore y' = a \Rightarrow y'' = 0$. Thus $y'' = 0$ is the required Differential Equation.

An equation involving derivatives of the dependent variable with respect to independent variable (variables) is called a differential equation. If there is only one independent variable, then we call it as an ordinary differential equation. For eg: $2 \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$.

It is the order of the highest order derivative occurring in the Differential Equation For eg: the order of $\frac{dy}{dx} = e^x$ is one and order of $\frac{d^2 y}{dx^2} + x = 0$ is two.

It is defined if the Differential Equations is a polynomial equation in its derivatives, and is defined as the highest power (positive integer only) of the highest order derivative.
 For eg: the degree of $\left(\frac{d^2 y}{dx^2}\right)^3 + \frac{dy}{dx} = 0$ is three
 Order and degree (if defined) of a D.E. are always positive integers.

A function which satisfies the given Differential Equation is called its solution. The solution which contains as many arbitrary constants as the order of the D.E. is called a general solution and the solution free from arbitrary constants is called particular solution.
 For eg: $y = e^x + 1$ is a solution of $y'' - y' = 0$. Since $y' = e^x$ and $y'' = e^x \Rightarrow y'' - y' = e^x - e^x = 0$.

Differential Equations

